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Technique for Direct Measurement of Skin Friction in High Enthalpy Impulsive Scramjet Flowfields

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A new technique was developed that allowed the direct measurement of skin friction on the walls of the inlet and combustion chamber associated with two scramjet models being tested in two hypersonic impulse facilities. The scramjet experiments took place in both the NASA Ames 16-in. high-enthalpy hypersonic and the CALSPAN 96-in. shock tunnels. Flight Mach number conditions of 12–16 were tested. Miniature, plastic, cantilever skin friction sensors were developed for the present tests. The use of plastic provided the necessary high-frequency response, high-shear sensitivity, and low-thermal sensitivity. The cantilever design was incorporated to minimize the gauge sensitivity to normal pressure. The gauge performance was found to be excellent for a variety of controlled evaluation experiments, where the results agreed very well with other independent direct measurements and accepted empirical correlations. The gauge also proved to be accurate and reliable in the two impulsive scramjet experiments.

Nomenclature

C_f = skin friction coefficient, τ_w/q
C_p = specific heat at constant pressure
h = convection heat transfer coefficient
M = Mach number
P, p = pressure
Pr = Prandtl number
q = dynamic pressure, $\frac{1}{2}\rho U^2$
q_w = wall heat flux
Re = Reynolds number
St = Stanton number
T = temperature
U = velocity
x, y = Cartesian coordinates
γ = ratio of specific heats
δ = boundary-layer thickness
ρ = density
τ = shear

Subscripts

aw = adiabatic wall
e = edge
FE = conventional floating element
i = incompressible
RA = Reynolds analogy
t = total condition
VD = Van Driest II theory
w = wall

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Introduction

IMPULSE facilities are currently used for ground testing for hypersonic flows at true flight conditions. Two Scramjet engine configurations were recently tested in the NASA Ames 16-in.[1] and CALSPAN Corporation 96-in.[2] shock tunnel facilities. Typical run times range from 0.5 ms to about 2.0 ms. These short test times provide the test engineer with the challenging task of instrumenting the models. Especially difficult is the measurement of skin friction. In fact, archived literature concerning direct measurement of skin friction in impulse facilities is extremely rare.

Direct measurement of skin friction is important for both practical and scientific reasons.[3] Scientifically, skin friction is critical for the proper scaling involved in the development of turbulence models. From a more practical perspective, skin friction plays a key role in the drag and, hence, the available thrust in a scramjet combustor. Indirect methods also exist for measuring skin friction, and these methods have been applied to impulse facilities. For example, heat flux measurements along with the assumed Reynolds analogy allows the skin friction to be inferred. Reference 4 presents a thorough review of indirect methods. Direct measurements are obviously preferred over indirect methods, since no assumptions are required.

The present gauge design was guided by the following criteria: high-frequency response, high wall shear force sensitivity, low weight (to minimize acceleration effects), and low-thermal and normal pressure sensitivities. Miniature axisymmetric vertical cantilever sensors (see Fig. 1) were constructed from ULTEM, LEXAN, and Victrex plastics.[5] The material properties of the plastic, the mechanical attributes of the cantilever geometry, and the usage of modern semiconductor strain gauges were used to accomplish all of the above mentioned desired skin friction gauge characteristics with a minimal amount of reliance on cancellation techniques. In addition, the floating element skin friction gauge design parameter criteria developed by Allen[6] were incorporated into all of the present gauges.

The baseline gauge was designed with a nominal natural frequency of 10 kHz. Thus, the expected time response was roughly 0.3 ms (approximately three times of the natural period). The gauges were designed to be sensitive to shear forces in the range of

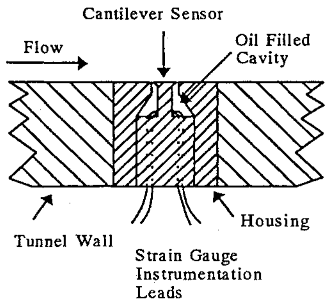


Fig. 1 Miniature cantilever beam skin friction gauge concept.

100–10,000 Pa. Analysis of the cantilever design indicated that the baseline sensor was approximately 60 times less sensitive to normal pressure as compared to shear forces. Heat transfer analysis indicates that the sensing elements at the base of the cantilever sensor will not encounter any temperature effects until after approximately 1000 ms (the extreme case of $\Delta T = 5000$ K and a wall heat flux of 3.0 kW/cm^2 was used in the analysis). Experimentally, the gauge has proven to be insensitive to the acceleration caused by the diaphragm bursting in the shock tunnel.

To assess the performance of the new gauge, a variety of controlled experiments were performed. The gauge was found to produce repeatable results that agreed with accepted experimental correlations. Since alternate techniques to measure skin friction in impulse facilities were not available, a direct comparison of the new gauge to another proven method was performed in a long duration blowdown facility. The new gauge produced results that agreed very well with those measured by a proven conventional floating element skin friction gauge. In addition, the CALSPAN inlet ramp data allowed for an additional gauge evaluation by comparing the direct measurement of skin friction to an estimate based on the Reynolds analogy and the measured mean heat flux. Excellent agreement was found.

Theoretical calculations of skin friction for three-dimensional, supersonic combustion, turbulent, viscous flows are subject to considerable uncertainty due mainly to turbulence modeling issues. The lack of accurate experimental data is a key reason for the current uncertainty in turbulence modeling. Skin friction has proven to be very important for proper scaling of turbulent boundary-layer flows. Thus, the present study addresses the issue of providing direct measurements of skin friction for flow conditions that for current ground testing can only be achieved in impulse facilities.

Description of the Gauge

Figure 1 demonstrates the basic concept of the new skin friction gauge. As can be seen, the wall shear force due to the flow displaced the head of a vertical cantilever beam. The beam displacement was monitored by strain gauges mounted on the base of the cantilever. The sensor was surrounded by a housing, and the cavity was filled with silicon oil.

The plastics used for the sensor and housing were ULTEM, LEXAN, or Victrex.[5] The baseline ULTEM gauge design is given in Fig. 2. The housing for the baseline gauge was arbitrarily chosen to be constructed from the same material as the gauge for the present study. Furthermore, the outer dimension of the housing corresponded to standard size of the material rod as received from the manufacturer. However, the shape and material of the outer housing can be designed to match experimental requirements, thus providing a relatively small and robust sensor. The baseline gauge was designed to provided a 10-kHz natural frequency. The design natural frequency was verified by experimental vibration analysis. The skin friction gauges were designed to measure shear forces as low as 100 Pa, or as large as 10,000, with a nominally linear response. A representative 1000-Pa wall shear displaced the baseline gauge head about 0.2 μm, which produced 20 microstrains. The intrusion of the head into the flow associated with this small displacement was essentially nonexistent, i.e., for the 1000-Pa wall shear, the head lip would protrude approximately 0.07 μm. Semiconductor strain gauges, mounted at the base of the cantilever, were employed to measure the relatively small values of strain. The axisymmetric cantilever design was chosen to minimize normal pressure sensitivity. Mechanically, the cantilever geometry of the baseline gauge was found to be 60 times less sensitive to normal pressure sensitivity as compared to the desired shear force. In addition, the symmetric beam allowed any remaining normal pressure signal to be electronically cancelled via the one-half-bridge arrangement of the strain gauges.

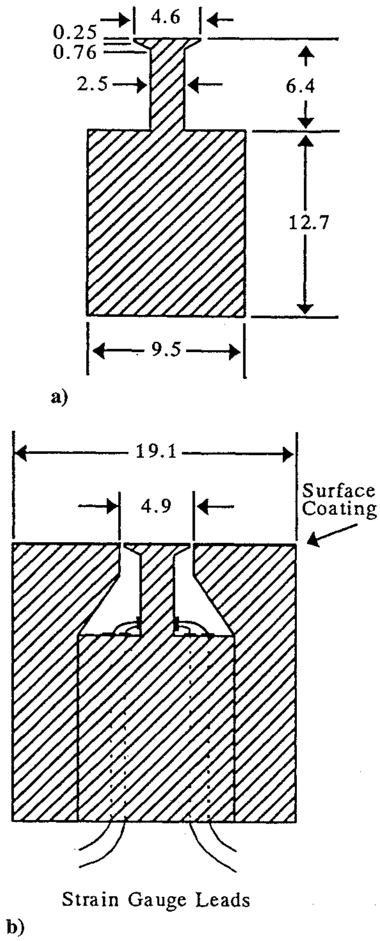


Fig. 2 Baseline 10-kHz ULTEM 1000 gauge design, mm: a) sensor and b) sensor and housing.

The cavity surrounding the sensor was filled with silicon oil. The oil had three purposes. First, the oil was incompressible, which minimized the errors due to pressure gradients along the head (i.e., ideally, the guage behaved as a solid wall). Second, the oil thermally isolated the temperature sensitive semiconductor strain gauges. Finally, the oil, coupled with the low specific gravity of the cantilever sensor, helped to damp the sensor vibrations due to the impulse facility starting shock. The gauge design parameters, e.g., gap to diameter ratio, lip size, etc., were chosen to minimize the possible errors as discussed by Allen.[6]

The thermal response was a very important criteria in the skin friction gauge development. A temperature difference of about 5000 K, i.e., a freestream total temperature of about 6000 K and 300 K starting wall temperature, and a wall heat flux of 3.0 kW/cm^2 would imply, based on a one-dimensional, semi-infinite, convection heat transfer solution,[7] that the temperature 2 mm below the surface increases only 1°C in approximately 1000 ms. Hence, even for these assumed harsh conditions, the strain gauges mounted near the base of the sensor should not be affected by temperature during a typical 0.5–2.0-ms run.

Variations of the baseline design were also studied. First, 0.02-μm metallic or ceramic coating was electronically deposited on to the sensor head, as well as the surrounding housing. This provided an aerodynamically smooth, durable guage surface. A second variation was to adhere a machinable ceramic head to the tip of the cantilever. This modification was implemented to increase the gauge durability in combustion flows. Both of these modifications were included to minimize the uncertainty due to surface blistering and scarring. Third, the cantilever beam was hollowed in order to decrease the impulse facility initial shock vibration response by reducing the mass.

Results

Gauge Evaluation Studies

The first gauge evaluation test was performed in a Mach 2.8 Ludwig tunnel, where $P_t = 14.5 \pm 0.15$ atm and $T_t = 295 \pm 2$ K, where the uncertainties were based on transducer calibrations. Both LEXAN and ULTEM 1000 gauges were tested. A flat plate model was located 2.5 cm downstream of the axisymmetric nozzle. Figure 3 presents the nozzle exit pitot pressure and the skin friction gauge traces. The pitot trace was acquired with a 330-kHz pressure transducer facing directly into the flow. The three lines on Fig. 3b labeled C_f gauge represent the wall shear results. The solid line results were obtained with a 5-kHz natural frequency LEXAN gauge, where the two dashed line traces correspond to the data measured with two different 10-kHz natural frequency ULTEM 1000 gauges. The steady test time marked on the figure (between 0.015 and 0.030 s) indicates the time between the end of the starting process and the arrival of the reflected expansion. The mean wall shear over this interval was measured as 1550 Pa. This value produced a skin friction coefficient of 0.0024. The mean results from the three gauges agreed to within about 3.0%. However, the noise content of the signal was estimated at 9.8%. The pitot trace in Fig. 3a corresponded to the same run as the solid line on skin friction trace, where upon careful examination, it was noticed that the peaks and valleys appeared to line up, which was taken as an indication that they were correlated and, hence, flow related. Also, perturbation analysis indicated that these nominal 3.6% variations in the pitot pressure induced a 3.3% scatter in dynamic pressure (based on a local Mach number of 1.7).

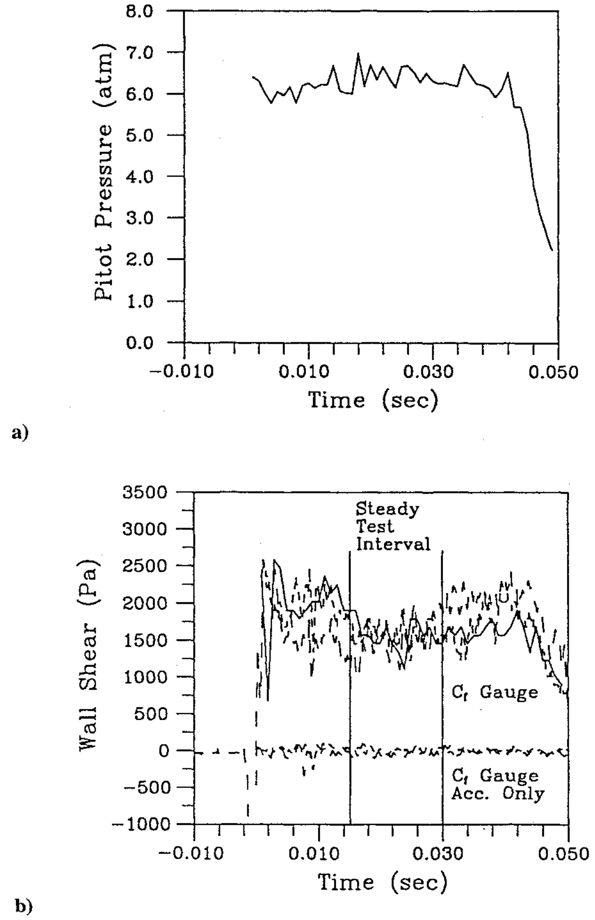


Fig. 3 Mach 1.7 Ludwig tunnel flat plate results: a) nozzle exit pitot pressure and b) wall shear and acceleration effect traces.

During the second two tests, two 10-kHz baseline gauges were used simultaneously. One was located such that it was exposed to the flow. However, the second gauge was mounted on to the model but not exposed to the flow. The purpose of this arrangement was to assess the response of the skin friction gauge to accelerations. The present facility was well suited for this, since it produced a somewhat harsh starting dynamic. The starting acceleration was measured at 100–200 g, with a recoil of nearly 1.0 cm. The skin friction gauge acceleration only results are also shown in Fig. 3b (labeled C_f Gauge Acc. Only). These results demonstrate that the gauge was very insensitive to the acceleration of the tunnel starting shock.

Since techniques to measure skin friction in impulse facilities were very rare, an empirical analysis was performed to estimate the gauge accuracy. The flat plate model was located downstream of the nozzle, which was operating at an overexpanded condition, a shock reflected about 0.5 cm downstream of the leading edge of the flat plate. The freestream Mach number local to the skin friction gauge was determined to be 1.7. The gauge was located 3.8 cm downstream from the leading edge. Based on these conditions and the Schoenherr empirical skin friction formula, the widely accepted Van Driest II correlation as shown in Schetz[8] was used to estimate the skin friction coefficient as 0.0022.

Since the Van Driest II theory involves some empiricism, its accuracy depends upon experimental data. Hopkins and Inouye[9] found the Van Driest II theory agreed with experimental data to within ±10%. This possible uncertainty, coupled with the expected experimental scatter of the measured flow properties, indicated a rather large uncertainty in the analysis. For example, a 4.0% uncertainty in the pitot pressure implied, based on a perturbation analysis, an additional 5.0–10.0% uncertainty in the skin friction prediction for $M \in [1.7, 3.0]$. Thus, the uncertainty associated with the Van Driest II analysis for the present gauge evaluation studies was roughly 15.0–20.0%. Hence, the agreement of the measured skin friction value with the present gauge was well within the uncertainty of the Van Driest II prediction. Table 1 summarizes the new gauge evaluation results obtained from the controlled experiments.

To compare the new gauge results to other directly measured skin friction values, baseline LEXAN and ULTEM gauges were tested in the Virginia Polytechnic Institute Mach 2 blowdown facility, where $P_t = 6.0 \pm 0.15$ atm and $T_t = 285 \pm 2$ K. Since this facility allowed long run times, proven lower time response techniques could be used, where the tunnel was operated for many minutes in order to obtain the C_f data. However, for the present gauge, the tunnel run times were limited to nominally 0.75 s. Figure 4a presents the ULTEM gauge wall shear trace. The tunnel on/off valve was a hand thrown gate valve, thus the erratic looking first 0.25 s corresponds to the valve opening dynamics. After roughly 0.35 additional seconds the valve was closed. As can be seen, the average wall shear over the steady test time was nominally 400 Pa, which indicated that $C_f = 0.0018$. Figure 4b shows the results of the LEXAN gauge, where the valve was pulsed on and off three times. As can be seen, the steady test intervals for the pulses were about 0.1 s in duration and the nominal wall shear stress was nominally 400 Pa for all three.

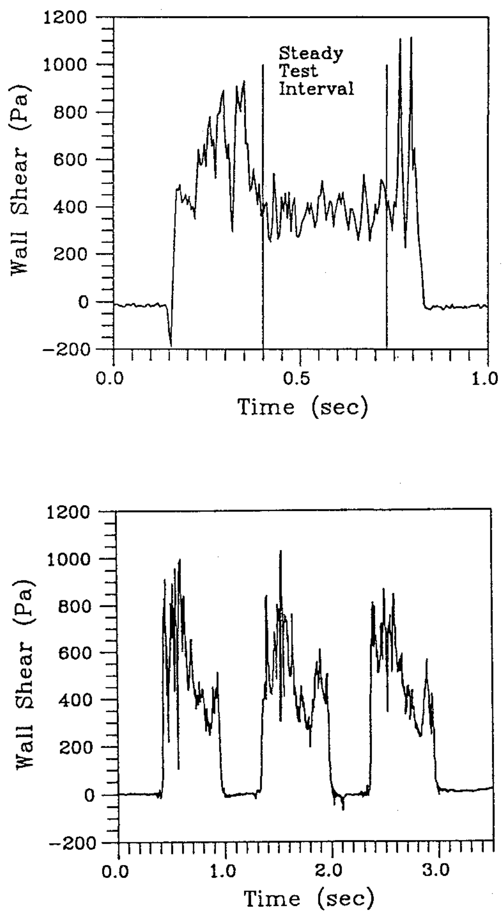
The mean value of C_f in this facility was found to be repeatable to within 10.0%. However, the noise content of the signal was 17.0%. The frequencies of the noise were much lower than the 10-kHz natural frequency of the sensor, which indicated that the noise was not related gauge vibrations. The skin friction for this facility has been well documented[10] at $C_f = 0.0020 \pm 0.0002$ by a conventional floating element gauge, such as those described Deturris et al.[3] The noise content of those signals were 12.0%.[10] The source of the tunnel noise is unclear. In any event, the new gauge measured a mean C_f that agreed to within the uncertainty of results of a proven technique. Van Driest II theory predicts a C_f of 0.0019, which was also in good agreement with the measurements. These results are also summarized in Table 1.

The last gauge evaluation experiment was performed in the Virginia Polytechnic Institute Mach 3 shock tunnel, where $P_t =$

Table 1 Summary of new skin friction gauge evaluation results

Run time, ms	Local Mach no.	P_t, atm	T_t, K	C_f	$C_f\|_{VD}$	$C_f\|_{FE^a}$	Figure no.
15	1.7	14.5	395	0.0024	0.0022	NA[b]	3
350	2.0	6.0	285	0.0018	0.0019	0.0020	4
5	3.0	9.0	620	0.0014	0.0016	NA	5

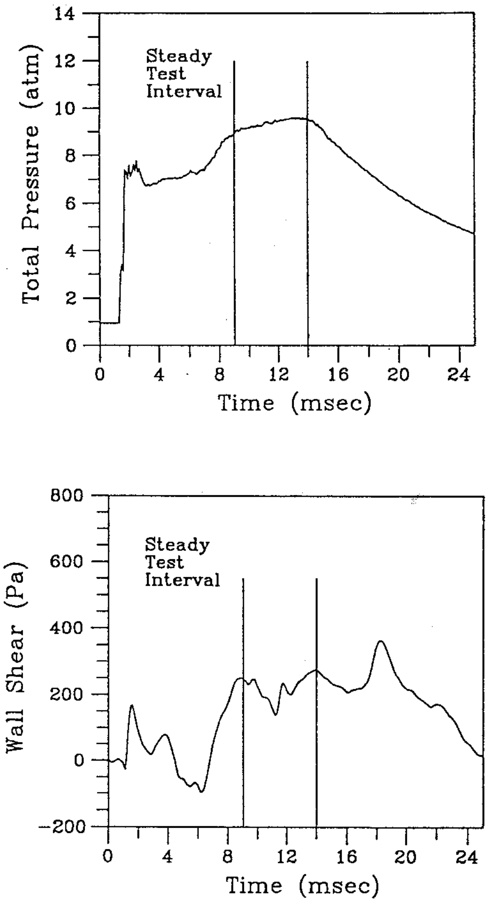
[a]Conventional floating element C_f results are only available for the long duration test; uncertainty of ± 0.0002.
[b]Not available.



a)

b)

Fig. 4 **Mach 2.0 blowdown tunnel wall shear results: a) ULTEM baseline gauge trace and b) LEXAN baseline gauged pulsed tunnel valve results.**



a)

b)

Fig. 5 **Mach 3 shock tunnel results: a) total pressure and b) wall shear trace.**

9.0 ± 0.15 atm and $T_t = 620$ K (computed). The total pressure and wall shear traces are given in Fig. 5. The 5.0-ms test time is indicated on both figures. These traces are typical for reflection-type shock tunnels. The spike in the pressure trace at 2.0 ms corresponds to when the incident starting shock struck the nozzle inlet. The pressure behind the reflected shock increased due to the length of the nozzle inlet until a steady nozzle starting pressure was achieved. Shadowgraph photography verified that the tunnel had not started until the pressure reached the plateau at about 8.5 ms. The unsteady starting phenomena is also noticeable in the first 8.0 ms of the wall shear trace. However, once the tunnel starts, the wall shear jumps to a nominally steady value of 220 Pa. The corresponding skin friction coefficient was 0.0014. Van Driest II theory (based on the measured boundary-layer thickness of 5 mm) and the flow conditions produced a C_f of about 0.0016. Again, the gauge results are within the expected 15.0–20.0% uncertainty of the analysis. These results are also included in Table 1.

Scramjet Experiments

Based on the successful gauge evaluation results, the new gauges were incorporated into two scramjet experiments being conducted at NASA Ames[1] and CALSPAN Corporation.[2] The model configurations and detailed flowfield data have not been released for publication. However, since the main focus of this paper is the measurement technique, this is not considered a limitation. Table 2 summarizes the measured wall shear and reported flight conditions from each facility.

Measurements were obtained in the inlet ramp and in the combustion chamber of the model being tested in the CALSPAN 96-in. shock tunnel.[2] Mach 12–14 enthalpy conditions were being tested. The first data set corresponds to a Mach 12 configuration.

Figure 6 presents an inlet wall shear trace. In this test, a 9.0-kHz ULTEM gauge with a hollowed beam was used. The gauge response to the shock tunnel starting characteristics are typical for reflection shock tunnels. It is important to note that in reflection type shock tunnels, the usable test time is a very small percentage of the total blowing-out of the tunnel gases (driver and driven). Hence, it is unlikely that the gauge traces would have returned to zero in the traces presented here. The steady test interval occurred between the shock reflection and the arrival of the driver gas (or contact surface). CALSPAN reported the 0.5-ms test interval.

The expected time response for this gauge was approximately 0.3 ms. As can be seen in Fig. 6, the wall shear trace levels off

Table 2 Summary of scramjet skin friction gauge results

Facility	Gauge location	Flight Mach no.	P_t,[a] atm	T_t,[a] K	τ_w, Pa	$\tau_w\|_{RA}$, Pa	$\tau_w\|_{VD}$, Pa	Figure no.
CALSPAN	Inlet	12	190	6900	850	820	1060	6
CALSPAN	Inlet	14	190	10000	1320	NA[b]	NA	7
CALSPAN	Combustor	12	27	3500	1000	NA	NA	8
NASA Ames	Inlet	16	NA	NA	450	NA	NA	9
NASA Ames	Combustor	16	NA	NA	600	NA	NA	10

[a]Computed from reported data.
[b]Not available.

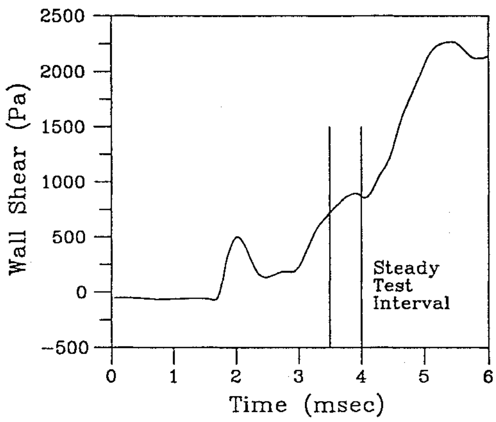


Fig. 6 CALSPAN scramjet inlet wall shear, Mach 12 flight conditions.

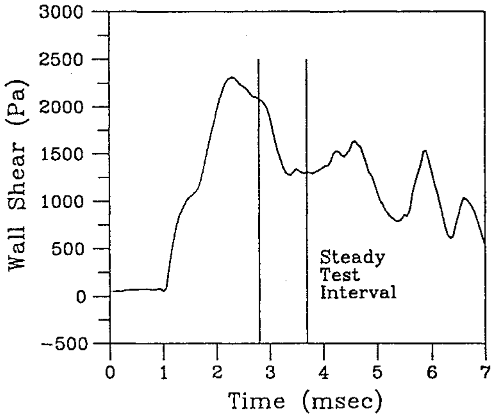


Fig. 7 CALSPAN scramjet inlet wall shear, Mach 14 flight conditions.

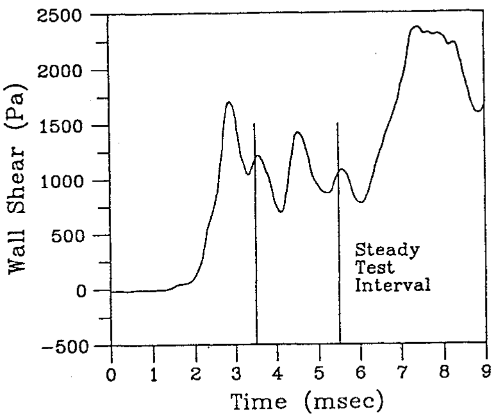


Fig. 8 CALSPAN scramjet combustor wall shear, Mach 12 flight conditions.

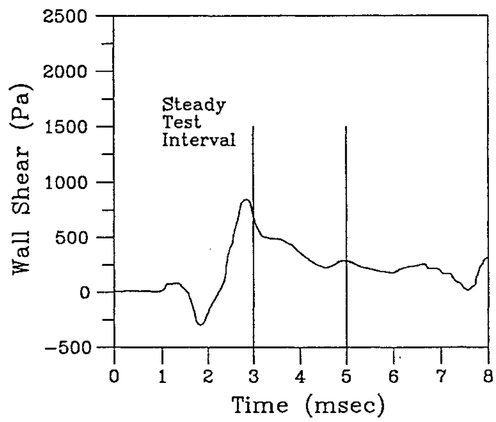


Fig. 9 NASA Ames scramjet inlet wall shear, Mach 16 conditions.

after about 0.3 ms at 900 Pa. An Re_δ of 89,000 and the Blasius empirical correlation[8] with the Van Driest II theory predicted that $\tau_w = 1060$ Pa. It was expected that the Van Driest II theory would overpredict the skin friction for the present cold wall flow.[11] Heat flux measurements were also obtained on the inlet ramp. The heat flux was measured as 230 W/cm^2. Assuming the Reynolds analogy ($C_f = 2StPr^{\frac{2}{3}}$) to be valid, τ_w was estimated as 820 Pa. Flowfield measurement uncertainty estimates for the present scramjet experiments were not available for publication. However, the agreement between the directly measured skin friction and the approximate results was considered excellent.

Figure 7 presents the inlet wall shear trace for a Mach 14 flight condition. The steady test time was again reported by CALSPAN. During the first 0.3–0.4 ms of the steady test interval, the gauge appears to be responding to the somewhat atypical starting dynamic. The average wall shear over the remaining test period was approximately 1320 Pa.

Figure 8 gives the shear trace measured on a wall in the combustion chamber. The reported test time is indicated on the wall shear trace. The shear trace appeared to vary in time, over the specified test interval, at a frequency of about 1.0 kHz. The average wall shear over the steady test period was approximately 970 ± 200 Pa (nominally 1 standard deviation). The cause of the oscillation was uncertain. However, it did not coincide with the natural frequency of the gauge, which implied that this phenomena was not related to gauge vibrations. The results of the CALSPAN experiments are summarized in Table 2.

Skin friction measurements were also obtained in the inlet ramp and combustion chamber in a scramjet engine model being tested in the NASA Ames 16-in. hypersonic high-enthalpy shock tunnel.[1] Figure 9 gives the wall shear trace obtained on the inlet cowl. Note, the gauge response to the tunnel starting was slightly different than that of the CALSPAN facility. Here, the gauge initially indicated a negative shear, followed by a jump to the shear level. This dynamic was also observed in some of the gauge evaluation tests. It is believed that this phenomena is related to the facility starting dynamics. The average wall shear over the test interval indicated in Fig. 9 was 450 Pa. The combustor wall shear stress was estimated from the trace given in Fig. 10 as 600 Pa. The results of the NASA Ames experiments are also summarized in Table 2.

As mentioned in the Gauge Description section, variations of the baseline gauge were also tested. In general, the hollowed or solid beam gauge design, with the metal or ceramic coated heads, were found to be the most reliable and robust. The gauges with adhered ceramic heads proved to be problematical. Head adhesion and gauge frequency response were the key issues. The bare plastic head gauges were found to suffer from surface erosion and scarring.

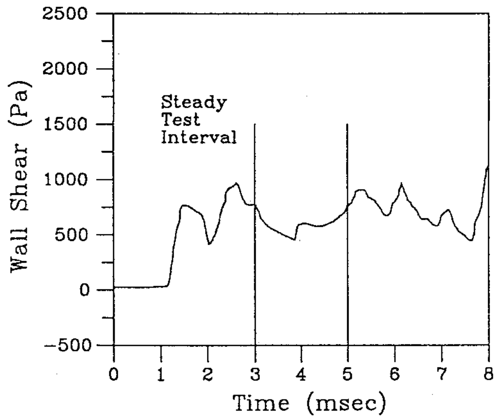


Fig. 10 NASA Ames scramjet combustor wall shear, Mach 16 conditions.

Conclusions

High-enthalpy impulse facilities are currently the only option for ground testing at true hypersonic flow conditions, and accurate knowledge of skin friction is critical for both theoretical turbulence modeling and practical drag estimation. A new technique for the direct measurements of skin friction in high-enthalpy impulsive flowfields has been developed here. The new gauge eliminates the necessity of relying on unnecessary assumptions such as the Reynolds analogy. A miniature cantilever design made from engineering plastic and instrumented with high-sensitivity semiconductor strain gauges met the skin friction gauge requirements of high-frequency response, high-shear sensitivity, low thermal sensitivity, and normal pressure and acceleration sensitivity. Based on the results from a series of controlled experiments, the gauge was found to be accurate, robust, and reliable.

The gauge was incorporated into two high-enthalpy scramjet experiments to measure the wall shear stress associated with the three-dimensional boundary layers associated in both the inlet and combustion chamber. Flight Mach number conditions of 12–16 were tested, where the total temperature and pressure were as high as 10,000 K and 190 atm, respectively. The new gauge has proven to be reliable under these very demanding conditions.

Acknowledgments

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